Closing next wed: HW_2A,2B,2C
Office Hours: 1:30-3:00pm in Com.B-006

Quick review:
Def'n: The "signed" area between $f(x)$ and the x-axis from $x = a$ to $x = b$ is the *definite integral*:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$

FTOC(1): Areas are antiderivatives!

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

FTOC(2): If F(x) is <u>any</u> antideriv. of $f(x)$,

$$\int_a^b f(x)dx = F(b) - F(a)$$

Entry Task: Evaluate

$$\int_0^4 e^x + \sqrt{x^3}\,dx$$

$$\int_3^6 \frac{4}{x} - \frac{2}{x^2}\,dx$$

5.4 The Indefinite Integral and Net/Total Change

Def'n: The **indefinite integral** of $f(x)$ is defined to be the general antiderivative of $f(x)$. And we write

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is any antiderivative of $f(x)$.

A brief pause to discuss current integration methods. We can currently find antiderivatives for sums and constant multiples of functions ***directly*** from our integration table.

Examples (we **can** currently do):

1. $\int 6e^x + 4x - 5\sqrt{x}\, dx$

2. $\int 6\sec^2(\mathrm{x}) - \frac{9}{x^4}\, dx$

Examples we **cannot** currently do (but will be able to do later in the term):

$$\int xe^{3x}dx\,; \qquad \int \tan(x)dx$$

$$\int x\sin(x^2)\,dx; \qquad \int \frac{x^2 - x + 6}{x^3 + 3x}dx$$

$$\int \frac{3}{x - 2\sqrt{x}}dx\,; \qquad \int \frac{\sqrt{x^2 - 1}}{x^2}dx$$

Examples we will *"never"* be able to do:

$$\int \mathrm{e}^{\mathrm{x}^2}dx\,; \int \sec(x^2)dx$$

Here are two that look bad but we can currently do them, why?

1. $\displaystyle\int \frac{\sqrt{\mathrm{x}} - 3\mathrm{x}}{\mathrm{x}}dx$

2. $\displaystyle\int \frac{\cos(x)}{1 - \cos^2(\mathrm{x})}dx$

What is the value of:

$$\int_{\pi/4}^{\pi/2} \frac{\cos(x)}{1 - \cos^2(\mathrm{x})}\, dx$$

Net Change and Total Change

Assume an object is moving along a straight line (up/down or left/right).

Let

$s(t)$ = 'location at time *t*'

$v(t)$ = 'velocity at time t'

pos. $v(t)$ means moving up/right

neg. $v(t)$ means moving down/left

The FTOC (part 2) says

$$\int_a^b v(t)dt = s(b) - s(a)$$

i.e.

'integral of velocity'= '**net change** in dist'

We also call this the *displacement*.

Thus, in general, the FTOC(2) says the **net change** in $f(x)$ from $x = a$ to $x = b$ is the integral of its **rate**. That is:

$$\int_a^b f'(t)dt = f(b) - f(a)$$

We define **total change** in dist. by

$$\int_a^b |v(t)|dt$$

which we compute by

1. Solving $v(t) = 0$ for t.

2. Splitting up the integral at these t values, dropping the absolute value and integrating separately.

3. Adding together as positive numbers.

Example: $v(t) = t^2 - 2t - 8$ ft/sec
Compute the total distance traveled from t = 1 to t = 6.